

# Exponential Models

Name \_\_\_\_\_ Date \_\_\_\_\_

Exponential functions occur in a wide variety of real-world settings. Today we take a look at a few of the most common

You have probably worked with the formula for *compound interest* before.

$$A = P \left( 1 + \frac{r}{n} \right)^{nt}$$

Do you remember what the different variables mean?

1. Suppose \$1000 is invested at 6% interest. How much will you have after 5 years if the interest is compounded
  - (a) annually ( $n = 1$ )
  - (b) quarterly ( $n = 4$ )
  - (c) monthly ( $n = 12$ )
  - (d) daily ( $n = 365$ )

After seeing these results, it is natural to ask what would happen if  $n \rightarrow \infty$ ? In fact you find that in that case, the amount after  $t$  years is given by the formula  $A = Pe^{rt}$ . This is called *continuous compounding*.

2. If the interest in Question 1 is compounded continuously, how much will you have after 5 years?
3. Suppose you invest \$3000 at 8% interest. How long will it take for the investment to double in value if the interest is compounded
  - (a) annually
  - (b) continuously

Mathematicians find it convenient to use the base  $e$  for other situations involving exponential growth or decay.

Exponential growth:  $y = Ce^{kt}$ ,  $k > 0$

Exponential decay:  $y = Ce^{kt}$ ,  $k < 0$

What do  $C$  and  $k$  mean?

4. A colony of bacteria is growing exponentially, with the number of cells growing every 3 hours.  
(a) Using the exponential growth equation  $y = Ce^{kt}$ ,  $k > 0$ , find the value of  $k$ .

(b) How long will it take for the bacterial colony to triple in size?

Exponential decay models are used in carbon dating, a technique used by archeologists to determine the age of discovered sites or artifacts.

5. There are two isotopes of carbon. The more common, carbon-12 is not radioactive, but the other, carbon-14, is radioactive with a half-life of 5600 years. Use this half-life, and the exponential decay equation  $y = Ce^{kt}$ ,  $k < 0$ , to find the value of  $k$  for carbon dating.
6. Traces of burned wood found with ancient stone tools in an archeological dig were examined and found to contain 1.67% of the original amount of carbon-14. Approximately how long ago was the wood burned?

In other applications, exponential functions appear as part of more complicated functions.

7. If boiling water at  $100^{\circ}\text{C}$  is allowed to cool in a room whose temperature is  $20^{\circ}\text{C}$ , then the temperature  $T$  after  $t$  minutes is given by  $T = 20 + 80e^{kt}$ .

(a) If the water cools to  $80^{\circ}\text{C}$  in 5 minutes, find the value of  $k$ .

(b) Find the time needed for the object to cool to  $50^{\circ}\text{C}$ .

Basic exponential growth models assume that there are no limits to the growth, but this is frequently not realistic. For bounded growth, logistic models are frequently used.

8. The population of deer in a state park is given by  $P = \frac{1000}{1 + 9e^{-0.2t}}$ .

(a) Use your graphing calculator to sketch a graph. Make sure you choose a suitable viewing window. Make a rough sketch in the space below.

(b) What is the equation of the horizontal asymptote? What is its meaning?

(c) After how many years will the deer population reach 600?